

Answer of Home Work Problems

a) $(y+x^2-c)(x+\log y-c)=0$ b) $(y-3x-c)(y-4x-c)=0$

c) $(\log y-x-c)\{e^x(y+x-1)-c\}=0$ d) $(2y+3x^2-c)(2y-x^2-c)=0$

e) $(y-e^x-c)(y+e^{-x}-c)=0$ f) $(2y-x^2-c)(2x-y^2-c)=0$

Solve the following differential Equations:
(Home work)

a) $p(p^2+xy) = p^2(x+y)$ b) $p^2-(a+b)p+ab=0$

c) $p(p+x) = y(x+y)$ d) $xyzp^2 - (x^2y^2)p - xy = 0$

e) $p^3 - p(p^2 + x^2 + y^2) + x^2y + xy^2 = 0$

f) $p^3 - (x^2+xy+y^2)p^2 + (x^3y + x^2y^2 + xy^3)p - x^3y^3 = 0$

g) $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$

h) $p^2 + 2py \cot x = y^2$

Ex: Solve $p^2 y^2 \cos^2 x - 2p xy \sin^2 x + y^2 - x^2 \sin^2 x = 0$

Ans: The given problem can be written as,

$$(py)^2 - 2(py)x \tan^2 x + (y^2 \sec^2 x - x^2 \tan^2 x) = 0$$

$$\text{or } py = \frac{2x \tan^2 x \pm \sqrt{4x^2 \tan^4 x - 4(y^2 \sec^2 x - x^2 \tan^2 x)}}{2}$$

$$\left[\text{If } ax^2 + bx + c = 0 \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right]$$

$$\text{or } py = x \tan^2 x \pm \sqrt{x^2 \tan^2 x (\tan^2 x + 1) - y^2 \sec^2 x}$$

$$\text{or } y \cdot \frac{dy}{dx} = x \tan \alpha + \sec \alpha \sqrt{x^2 \tan^2 \alpha - y^2}$$

$$\text{or } y dy - x \tan \alpha dx = \pm \sec \alpha \sqrt{x^2 \tan^2 \alpha - y^2} dx$$

$$\text{or } \pm \frac{x \tan \alpha dx - y dy}{\sqrt{x^2 \tan^2 \alpha - y^2}} = -\sec \alpha dx$$

Integrating we get,

$$\pm \sqrt{x^2 \tan^2 \alpha - y^2} = c - x \sec \alpha, \text{ where } c \text{ being an arbitrary constant.}$$

Squaring we get,

$$x^2 \tan^2 \alpha - y^2 = c^2 - 2cx \sec \alpha + x^2 \sec^2 \alpha$$

or $x^2 + y^2 - 2cx \sec \alpha + c^2 = 0$ is the required solution.

Ex: Using the above method solve:

$$y \left\{ x(2x+1) \frac{dy}{dx} - y \left(\frac{dy}{dx} \right)^2 \right\} = 2x^3.$$

(Home work)